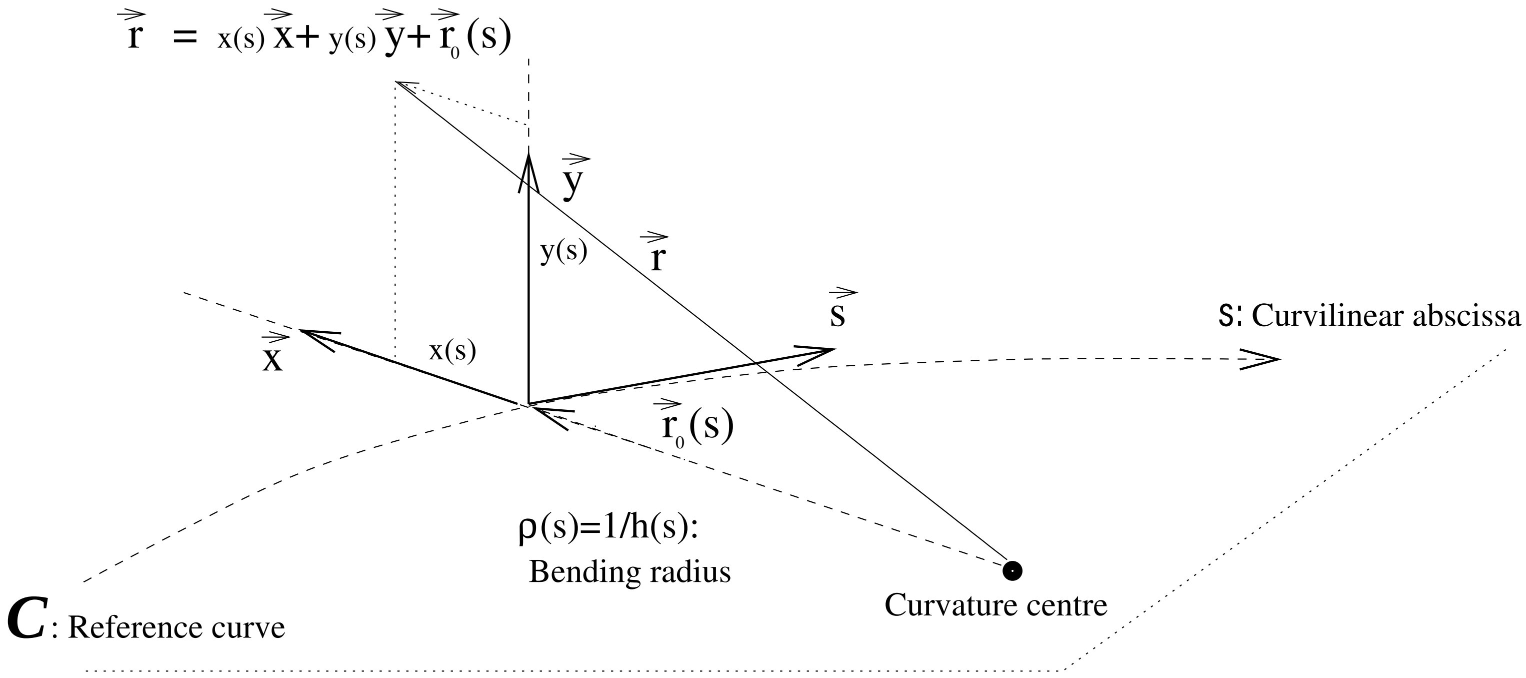
• *Referential for Ring1*: right-hand referential $(\vec{x}, \vec{y}, \vec{s})$, with $\vec{x}$ pointing **outward** and the curvature h positive since beam1 is deflected on the **right** in the plane oriented by $\vec{y}$.



• *Referential for Ring2*: right-hand referential $(\vec{x_2}, \vec{y_2}, \vec{s_2})$, with $\vec{x_2} \equiv -\vec{x}$ pointing **inward**, $\vec{y_2} \equiv \vec{y}$, $\vec{s_2} \equiv -\vec{s}$, and the curvature $h_2 \equiv -h$ negative since beam2 is deflected on the **left** in the plane oriented by $\vec{y}$.

Physical maps for Beam2 (for anti-clockwise tracking)

- In the **right-hand** referential $(\vec{x_2}, \vec{y_2}, \vec{s_2})$, the Hamiltonian for beam2 writes:

$$H\,(x, p_x, y, p_y, z, \delta) = -(1 + h_2\, x)\times$$
$$\left[\sqrt{\left(\frac{1}{\beta_0} + \delta - \frac{q V^{(2)}(x,y)}{p_0 c}\right)^2 - \frac{1}{\beta_0^2 \gamma_0^2} - \left(p_x - \frac{q A_x^{(2)}(x,y)}{p_0 c}\right)^2 - \left(p_y - \frac{q A_y^{(2)}(x,y)}{p_0 c}\right)^2} + \frac{q A_s^{(2)}(x,y)}{p_0 c}\right] + \frac{\delta}{\beta_0} \tag{1}$$

with $(V^{(2)}, \vec{A}^{(2)})$ the electromagnetic potential given in the referential $(\vec{x_2}, \vec{y_2}, \vec{s_2})$.
By convention, the electromagnetic field (main field and errors) is always defined in the referential $(\vec{x}, \vec{y}, \vec{s})$ of Ring1, so that

$$\left\{\begin{array}{rcr} V^{(2)}(x,y) & = & V(-x,y) \\ A_x^{(2)}(x,y) & = & -A_x(-x,y) \\ A_y^{(2)}(x,y) & = & A_y(-x,y) \\ A_s^{(2)}(x,y) & = & -A_s(-x,y) \end{array}\right. \tag{2}$$

- In the **left-hand** referential $(\vec{x} \equiv -\vec{x}_2, \vec{y}, \vec{s_2})$, the Hamiltonian for beam2 writes:

$$\tilde{H}\,(x, p_x, y, p_y, z, \delta) = H\,(-x, -p_x, y, p_y, z, \delta) = -(1 - h_2\, x)\times$$

$$\left[\sqrt{\left(\frac{1}{\beta_0} + \delta - \frac{qV^{(2)}(-x,\,y)}{p_0\,c}\right)^2 - \frac{1}{\beta_0^2\gamma_0^2} - \left(-p_x - \frac{qA_x^{(2)}(-x,\,y)}{p_0\,c}\right)^2 - \left(p_y - \frac{qA_y^{(2)}(-x,\,y)}{p_0\,c}\right)^2} + \frac{qA_s^{(2)}(-x,\,y)}{p_0\,c}\right] + \frac{\delta}{\beta_0} = -(1 + h\,x)\times$$

$$\left[\sqrt{\left(\frac{1}{\beta_0} + \delta - \frac{qV(x,\,y)}{p_0\,c}\right)^2 - \frac{1}{\beta_0^2\gamma_0^2} - \left(p_x - \frac{qA_x(x,\,y)}{p_0\,c}\right)^2 - \left(p_y - \frac{qA_y(x,\,y)}{p_0\,c}\right)^2} - \frac{qA_s(x,\,y)}{p_0\,c}\right] + \frac{\delta}{\beta_0}$$

The sign rules for the **anti-clockwise** tracking of beam2 are

$$\begin{cases} h_2 \longrightarrow -h_2 = h > 0 & \text{as for the MB's of Ring 1} \\ A_s \longrightarrow -A_s & \Rightarrow \text{ the multipole harmonics } (n \geq 1) \text{ change sign} \\ A_{x,y} \longrightarrow A_{x,y} & \Rightarrow \text{ the solenoid field does not change sign} \\ V_{x,y} \longrightarrow V & \Rightarrow \text{ the electrostatic separator field does not change sign} \end{cases}$$

These rules stand only for **“symmetrical magnets”** (invariant field under the transformation $s \longrightarrow L - s$) $\Rightarrow$ problems occur for dipoles with different entry and exit pole face rotations or for magnets with misalignment (essentially pitch and yaw angles).

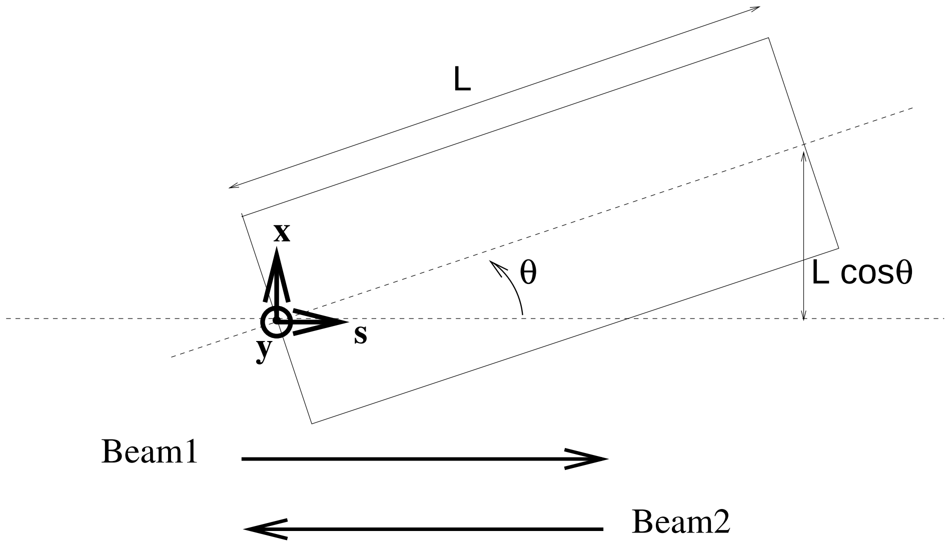
$\Rightarrow$ A `reflect` flag should be included in order to

1. Exchange the dipole pole face rotation for a tracking anti-clockwise.

2. Take a special care to the misalignment definition for the reflected sequence.

Example:

$\longrightarrow$ A yaw angle θ (rotation around $\vec{y}$) defined in lhcb2 clockwise corresponds, for lhcb2 anti-clockwise, to the misalignment $(dx = -L\cos(\theta)\,,\ -\theta)$.



$\longrightarrow$ More generally, the MAD8 "misalignment maps" $\mathcal{M}_{\rm in}$ and $\mathcal{M}_{\rm out}$ for beam1 clockwise becomes the reflected maps $\bar{\mathcal{M}}_{\rm out}$ and $\bar{\mathcal{M}}_{\rm in}$, respectively for beam2 anti-clockwise.

Un-physical maps for Beam2 (for clockwise matching)

The idea is to deal with the matching of Ring2 without reflecting the sequence lhcb2.
For this purpose, the so-called **reflected maps** must be used, consisting in

1. the inversion of the transfer map.

2. the change of sign of the variable p_x, p_y and t.

In the most general case, the $\bar{R}$ and $\bar{T}$ matrices of such a map can be deduced from the physical map (transfer map) by:

$$\bar{R}_{ij} = s_i s_j \left(R^{-1}\right)_{ij} \text{ and } \bar{T}_{ijk} = s_i s_j s_k \left(T^{-1}\right)_{ijk}$$

$$\text{with} \left\{ \begin{array}{l} s_1 = s_3 = s_6 = 1\,,\ s_2 = s_4 = s_5 = -1 \\ \left(T^{-1}\right)_{ijk} \stackrel{\text{def}}{=} -\sum_{l=1}^{6}\sum_{m=1}^{6}\sum_{n=1}^{6} R^{-1}_{il} T_{lmn} R^{-1}_{mj} R^{-1}_{nk} \end{array} \right.$$

In the case of (aligned or misaligned) multipoles or electrostatic separator, it can be shown that maps and reflected maps are the same.

For the solenoid case, it can be shown that the reflected map deduces from the transfer map by changing the sign of the actual B-field.